USING STRUCTURED APPARATUS

Dorothy Latham describes how the *Stern* apparatus can help to achieve the goals of the renewed primary mathematics framework.

In the renewed *Primary Framework for Mathematics* for England, great emphasis is given to calculation and its prerequisites (DfES, 2006). Expectations are increased for calculations and the recall of number facts, with mental calculation owning a higher profile, while progression in written calculation is clarified. Age-related expectations for learning multiplication tables have been brought forward, and a more carefully sequenced progression through the designated strands is to be supported by assessment.

Of the seven strands of mathematics laid out in the renewed framework, three central to all, deal with the increased emphasis upon calculation:

- counting and understanding number – counting, comparing and ordering numbers and describing relationships between them
- knowing and using number facts – having secure knowledge of number facts which can be recalled *quickly* and used and applied appropriately, together with a good understanding of the four operations
- calculating – being able to calculate mentally, both efficiently and accurately.

The greater focus on efficient and accurate calculation, based on knowing and understanding number bonds and operations, cardinal and ordinal aspects of number and the decimal number system itself, makes for certain priorities:

- a comprehensive understanding of concepts and processes, involving a carefully sequenced step-by-step progression of learning
- thoroughness of learning to ensure completeness of mental storage and retrieval; and
- sufficient frequency of practice to ensure speedy recall.

The involvement of working memory in thinking

When processing information or solving problems, we rely on working memory (Baddeley, 1997; Pickering, 2006). This acts as the essential mechanism for ongoing thinking and immediate organisation, but working memory is also a bit of a bottleneck in the flow of mental processing, because capacity is small and input fades fast. To pass information to the long-term memory where storage takes place, information has to be meaningful and often needs frequent repetition, although links between old and new information facilitate the process. Despite these limitations in working memory, recall from long-term memory can release more time and space for the ongoing cognitive operation; hence the importance of secure and accurate learning.

To summarise, the discovered features of working memory are:

- a capacity limit of 7 plus-or-minus 2 items for adults (3 to 6 for primary age children)
- fast fade, generally in a few seconds
- it deals with visual input in a visual mode, aural input in an auditory mode and so on, so that visual, auditory and kinæsthetic (VAK) imagery remains linked to meanings when stored.

Learning all number bonds surely and securely, with quick response, takes a very long time. It also needs carefully sequenced progression and frequent assessment to establish how far children have internalised number knowledge and understanding, before they can move on to the next part of the journey. I welcome the restated intention in the renewed framework that teachers need to look at

not only the age-related expectations for their classes but also those for the next and previous terms and years.

The rationale for the use of structured apparatus

How can we secure the knowledge, understanding and learning of all the processes and of the number bonds efficiently for our pupils? In order to provide for the visualisation, interest and manipulation which support learning, the use of some form of structured apparatus is indicated. There are different systems on the market, but not all include the same range of features, or are comprehensive in the requirements they fulfil. Such structured materials are described by Bristow et al (1999) as a bridge into knowing and understanding number. Bristow et al mention both the *Stern* apparatus and its derivative, *Numicon*, as being effective, because of their emphasis on number relationships and how they focus on these before the formal use of conventional notation is introduced.

Of the several structural systems I have used, the *Stern* apparatus struck me as the most comprehensive, the most essentially mathematical in concept, and having the most careful guidance (Stern, 2004). It was also the only one that had three-quarter inch modules, so designed by Dr. Catherine Stern to promote and capitalise on sensori-motor development, while (apart from *Unifix*, on a similar module) all the others used centimetre modules with no graded marks. I felt that for small children the larger-sized blocks and cubes would be much easier to handle. I also liked the number trays, making all the addition and subtraction facts a self-corrective activity, using their reasoning abilities to find the appropriate partner for each block in the stair (*figure 1*).

Stern also had the number track, now commonplace, and the 20-tray; a feature I still have not come across in any other product except for *Numicon*, a smaller version. This is invaluable for making the compositions of teen numbers and decades (*figure 2*). Finally, I liked the idea of the

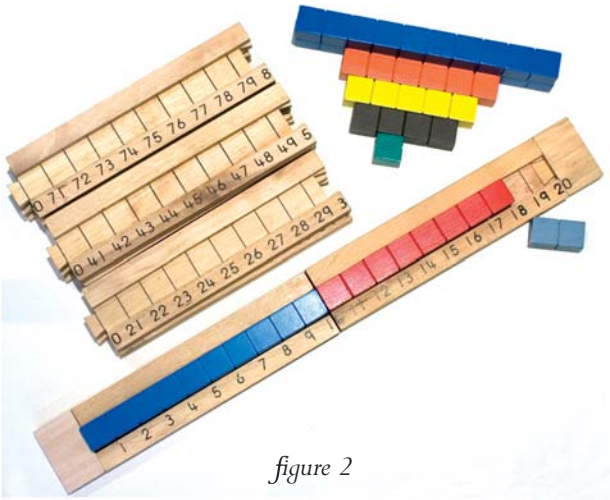
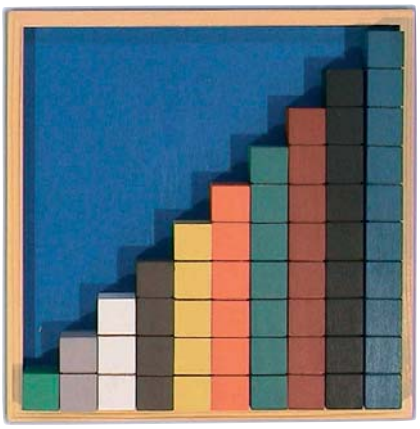
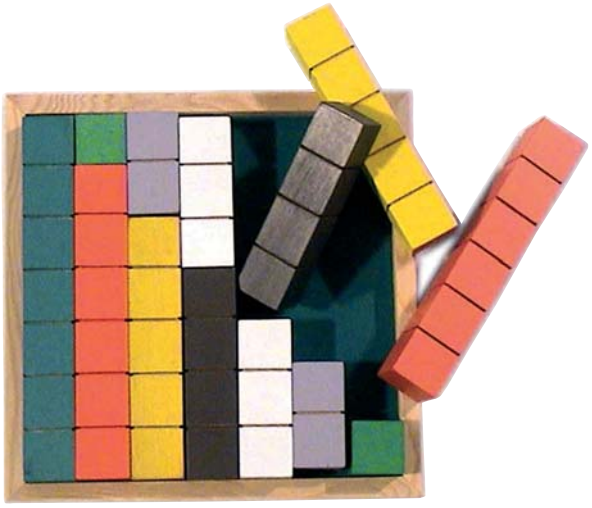


figure 2

figure 1





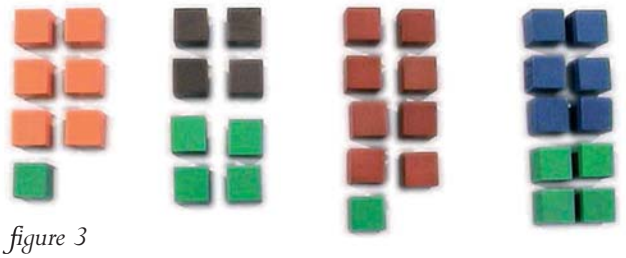


figure 3

pattern boards, to be used with cubes, providing experience of the ordinal aspect of numbers, to be combined with the cardinal values already displayed by the blocks (*figure 3*). Here, cubes in different colour arrangements are used to overcome any specific linkage of colour with number. While the colour of the blocks initially aids recognition, later, children need to understand that colour is not a fixed property of a number. *Colour Factor*, on the other hand, has rods and cubes based on the 1 cm module, with the colours specifically designed to aid recognition of number relationships and factors – these are similar in other ways to the centimetre module materials of *Cuisenaire*.

When I equipped both of my schools with the *Stern* apparatus, I also purchased extra sets of ten-blocks, useful for tens and units work, as well as '100-flats' (solid squares equivalent spatially to 10×10 blocks forming a 100 square) and the giant '1000-cubes' (*figure 4*). With these, h.t.u and th.h.t.u could be demonstrated easily and effectively, helping children grasp the whole decimal system. We also used the *Stern* mini sets of the blocks, exactly like the completed ten-trays, but based, like *Numicon*, on a 1cm module. For the older children, these took up less space on the desks. *Numicon* provide centimetre-sized rods and trays, including a 20-tray, and the Base Ten apparatus co-ordinates with these when big number work is embarked upon, having also '100-flats' and '1000-cubes' in its range. However, sufficient material to make up the '100-flats' is provided in *Stern Kit B*, enabling demonstration of values in hundreds, tens and units.

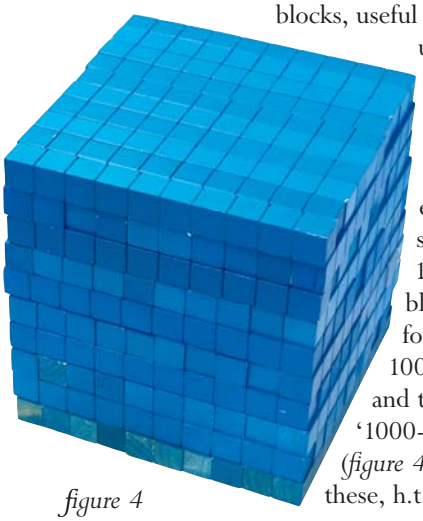
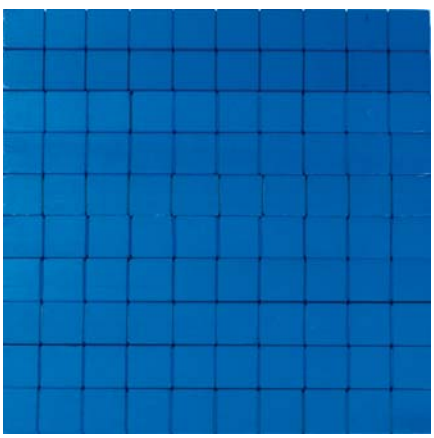


figure 4



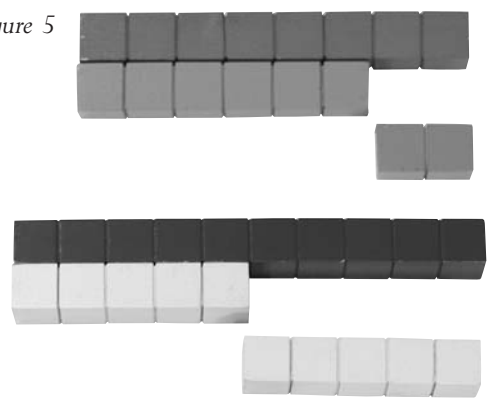
Some experiences

Because of the facilities afforded by the *Stern* apparatus, both ordinal and cardinal aspects of number are well exercised. Using the apparatus for the recommended activities focuses attention on the relationships between numbers. This was one of the most important principles in the design of the materials; when used in activities, they are able to display multiple relationships, often within a single view, and therefore in the consciousness of children. This starts from the recognition, construction and sequencing of series of patterns even at the stage when number names are first linked to blocks and patterns of cubes. Awareness of features such as odd and even numbers soon follows; for example, looking at the pattern boards, 'Is it a number with an odd one out who hasn't got a partner?' (see *figure 3*). Addition and subtraction of ones and of twos and so on, and the use of multiples in the number track, develop the understanding of relationships still further; for instance, of odds and evens, or of how many twos, or how many threes, or fives are contained in a number. This all builds the foundation for later understanding of prime numbers and factorisation, linking also to multiplication tables; eg, if we put six 4-blocks in the track, we can see the end of the line of blocks reaches the 24 mark.

In the infant years, we followed the suggested activities for the *Stern* apparatus, using the blocks for games first, without the use of number names. For instance, the 'feely bag game', where the teacher and children sit in a circle and pass round the feely bag, each taking one of the ten blocks from inside and hiding this behind their backs. They then build a stair together with the teacher putting down a ten (or a one) and saying "Who has the block that goes next?" and so on. Later they progress through various activities, such as the

'skyscraper game', where blocks 1 to 10 are randomly arranged in the 10-tray, forming a 'New York skyline'. The teacher then passes the tray around asking each child to complete a row with the complementary block (3 to 7 to make 10, 6 to 4 to make 10, and so on), leading to the filling of the 10-tray. After introducing number names with the blocks and pattern boards, we embarked on demonstrating the 'addition stories' of 10, 9, 8, 7, 6, 5, 4, 3, 2, 1. Children loved it – there was much merriment when the single story of one was reached each time! After showing them a few times, they were asked to record the stories themselves, at first just for the 10-tray and then subsequently for the others. When this became easy, it was for a while a daily task to write the addition stories of all the trays, 10 down to 1, as quickly as possible. After a while, different children would say spontaneously, "I don't need the blocks, I can do it from my head!" No fingers were allowed, but we made sure no one felt awkward if they still wanted to use the blocks – the important thing was to get it all right, and to do it quite quickly. All this did take time for practice, varying from child-to-child, but probably between one and three weeks for most. Some teachers were inclined to try to push children on who hadn't arrived at a sure enough internalisation and needed gently restraining. This is important. Although it does take this length of time, time is actually saved later on, because the grounding is so good, that later steps in learning are absorbed much more quickly. After learning all the addition facts to ten, next came learning of minus facts to ten. In fact, the renewed mathematics framework deals with the addition facts to five followed by the minus facts to five in order to emphasise the relational aspects of the two processes. In using the *Stern* apparatus, since children are taught to start by building a stair, it seems more logical and less cumbersome to go through the whole range of number facts from ten to one, as this emphasises the relationships both within and between the set of plus and minus operations. As well as learning the bonds, it is also crucial for children to become familiar with the multiple sub-concepts for both addition and subtraction, and their name labels, eg, count on, find the difference and so on (the latter is so easy with *Stern* blocks (*figure 5*).

figure 5



Following the acquisition of all the addition and subtraction facts to 10, which total 120 in all, or 132 with all zero facts included, we worked on the 20-tray to achieve the same mastery over the composition of teen numbers and 20 (See *figure 2*). Place value became important here, and for this the dual board is invaluable, with spaces for 10-blocks to indicate the 10s part of a number (on the left) and, on the right, a groove for units with the last space signalled, reminding the user to change 10 cubes for a whole 10-block when adding (*figure 6*). For subtraction, using decomposition it is obvious

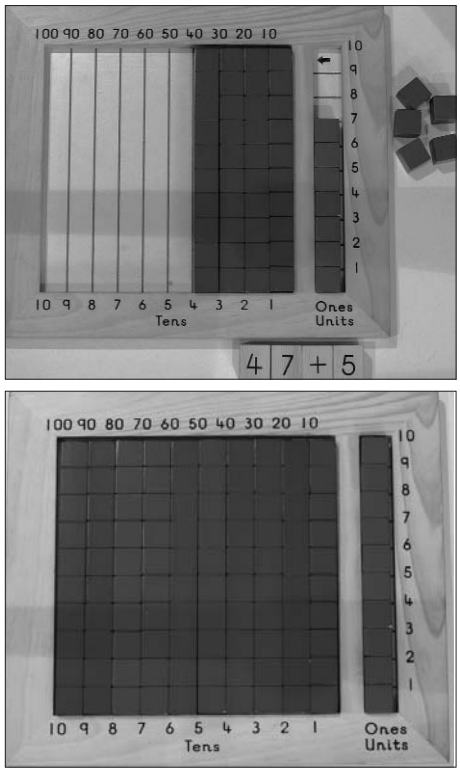


figure 6

that a 10-block must be changed for 10 cubes, placed in the groove, and then the relevant amount can be taken away. *Numicon* have a version of this on paper or card, but, again, with the actual grooves into which to fit blocks and cubes in the dual board, the sensori-motor aspect is being reinforced. Of course, we also used a number of other resources for reinforcing place value, such as bundles of sticks and single sticks, different sized rocks and pebbles and a variety of other commercial

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Structural maths apparatus

Cuisenaire and Base Ten, both from Learning Resources Ltd, 5 Merchants Close, Oldmeadow Road, King's Lynn, Norfolk PE30 4JX www.learningresources.co.uk

Numicon, from Numicon Ltd, 12 Pine Close, Avis Way, Newhaven, East Sussex BN9 0DH www.numicon.com

Stern Structural Apparatus, from Maths Extra Ltd, 3 North Street, Mere, Wiltshire BA12 6HH www.mathsextra.com

Unifix, from Philip and Tacey Ltd, North Way, Andover, Hants SP10 5BA www.philipandtacey.co.uk

materials in different forms, but the dual board we found essential, because it shows the processes of composition and decomposition across two-digit numbers so clearly.

Meanwhile we gained familiarity with numbers over 20 by building numbers in the number track, where any blocks could be 'added together' to make bigger numbers, or 10-blocks could be chained and added to units to demonstrate the tens and units of a number between 10 and 100 (eg, a row of three 10-blocks and five cubes shows 35) and, finally, we used 10-blocks and flats to work on operations with h.t.u., and later th.h.t.u. For instance, putting out two flats, four 10-blocks and three cubes to show 243.

Catering for differing abilities

As an example, I once gave a Y6 lesson, and using the mini *Stern* blocks (1 cm module) I aimed to teach pupils how to find the square of a number and the square root of a number, and to understand the concepts of squaring and square roots. I showed pupils how to start by building the square of a chosen number, say six, so that there would be six rows each totalling six units, to count the total number of units in the square, reaching the result of 36 for the square of six. After this, pupils worked in pairs setting square and square root discovery tasks for their partners with different numbers between one and nine. We then moved on to calculating square numbers and cube roots, building cubed structures. They all recorded what they did, and all the pupils' work was correct. Due to actually seeing and building the squares and cubes, they had effectively understood the necessary concepts and were also able to relate these to their known multiplication tables.

The structural apparatus of *Stern Structural Arithmetic* is a valuable aid for success in achieving the goals set for teaching and learning in the renewed framework due to its tactile and visual nature, its module size, its comprehensiveness, its careful sequencing and the virtually self-correcting nature of the items which comprise the sets, which are designed to promote reasoning. All of this supports the learning of basic number knowledge, leading to internalisation and speedy recall of facts, which in turn produces quick, efficient and accurate mental calculation, allowing maximum capacity for thinking about uses, applications and problems.

Dorothy Latham is an Independent Primary Education Consultant.

FOOTNOTE: REPORT ON USING *STERN* APPARATUS AS A SPECIAL NEEDS RESOURCE

'John' is a Y3 child who has been diagnosed as having moderate learning difficulties. He has a statement of special needs and is educated one year out of cohort (now in Y2), having repeated his year in nursery. He has hearing and speech and language difficulties and receives a high level of support (25 hours per week). Despite this level of carefully targeted support, using a range of materials and being in classes with very experienced and skilled teachers, John had entered the Y2 class still unable to count with 1-1 matching and with no real understanding of number values. In June 2006, towards the end of his time in Y1 (but as a Y2 child), it was decided to use the *Stern* materials to support John. The progress he has since made has amazed and greatly pleased all involved.

As a visual learner John quickly learnt to match the blocks to their home space. The multi-sensory nature of the programme helped to motivate him and make him actively involved in the learning process. The small steps with lots of repetition helped to lay solid foundations for understanding what number values really are, enabling him then to move on through other concepts. Having the spoken concepts linked to visual actions has helped to develop John's receptive language and auditory memory. He has particularly enjoyed taking the role of the teacher, where he needs to be able to give instructions and use expressive language appropriately. The layout of the pattern board reinforced accurate 1-1 counting and directionality and this has transferred to counting everyday objects.

John's learning accelerated through the next six-week period allowed on his individual education plan and he was able to work on concepts of one more, one less and simple number bonds to 10. This work was revisited after the summer holidays and by October 2006 John was working on simple addition and subtraction and demonstrated a very secure understanding of these concepts.

Now in March 2007, and within a nine-month period, John has gone from having no understanding of number values to being able to add and subtract and to record his work confidently.

Gaynor O'Ryan is Special Educational Needs Co-ordinator and Nursery Teacher at Holy Trinity CE Dobcross Primary School, Oldham.

Please note that the name of the child has been changed.

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